A7: Discovery of Exosolar Planets

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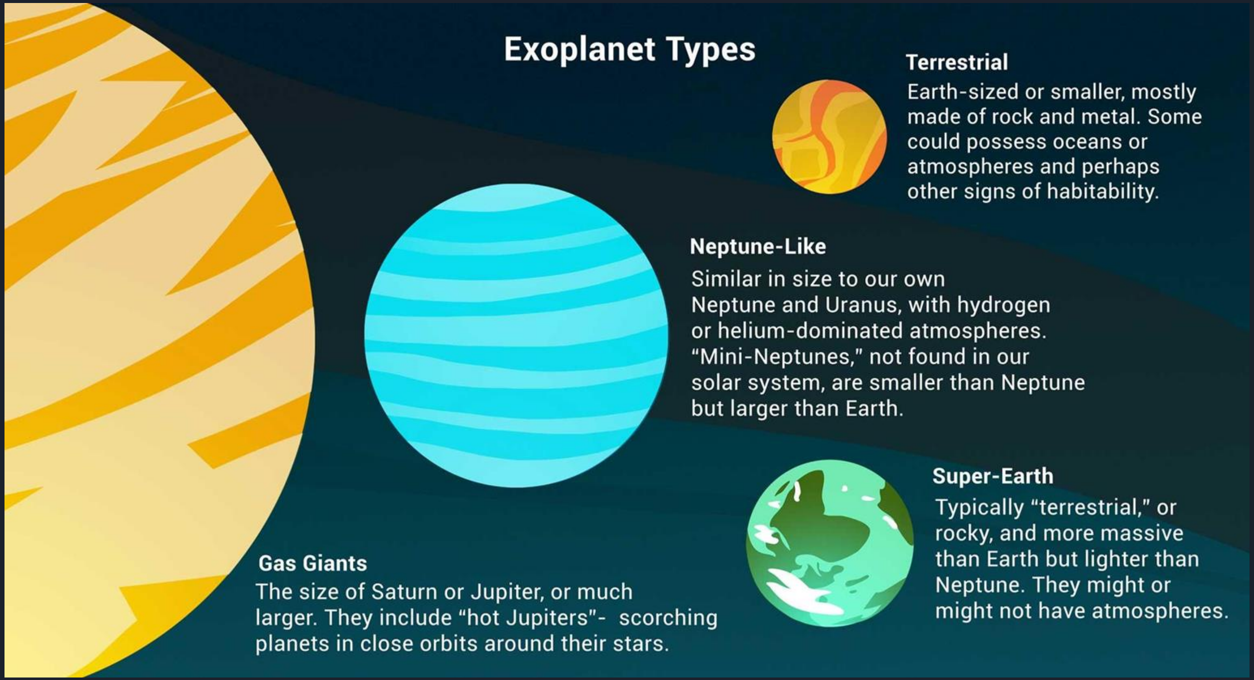
Background

- “Exosolar”
  - Beyond our solar system

- An exoplanet is any planet beyond our solar system.
  - Most orbit other stars
  - Some free-floating exoplanets, known as rogue planets, orbit the galactic center and are untethered to any star
  - *(Credit: NASA)*

Background



- *Credit: NASA/JPL-Caltech*

Why search for exoplanet

- One big question: **Are we alone in the universe?**
  - Are there solar systems or earth-like planets like ours
  - Habitability
  - Extraterrastial lifes?

Background

- The Kepler mission
  - Determine the frequency of Earth-sized planets in and near the habitable zone of Sun-like stars, within the Milky Way *(Credit: Borucki et al., 2010)*
  - The Kepler space telescope is a big photometer that continually monitored the brightness of stars
  - Observed 530,506 stars and detected 2,662 planets *(Credit: Wiki)*

- Habitable Zone
  - Distance from a star which liquid water could exist on surfaces of orbiting planets (*Credit: NASA)*
  - Brighter stars -> Further and larger zone, vice versa

Methods for exoplanet discovery

- Direct method
  - Direct imaging
- Popular indirect methods
  - **Transit timing**
  - Doppler spectroscopy / Radial velocity method

Light Curve

- When Planet transit, observed brightness of the star decrease
    - the decrease depend on size and distance between the star and planet

Transit model - Uniform Source

- d is the center-to-center distance between the star and the planet, r_P is the radius of the planet, r^* is the stellar radius, $z = d/r^*$ is the normalized separation of the centers, and $p = r_P / r^*$ is the size ratio
- the ratio of obscured to unobscured flux is $F = 1 - \lambda^e$.

$$\lambda^e(p, z) = \begin{cases} 0, & 1 + p < z, \\ \frac{1}{\pi}\left[p^2\kappa_0 + \kappa_1 - \sqrt{\frac{4z^2 - (1 + z^2 - p^2)^2}{4}}\right], & |1 - p| < z \leq 1 + p, \\ p^2, & z \leq 1 - p, \\ 1, & z \leq p - 1, \end{cases}$$

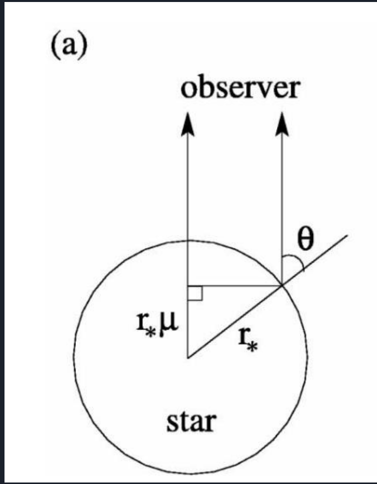
where $\kappa_0 = \cos^{-1}[(1 + p^2 + z^2)/2z]$, $\cos^{-1}[(p^2 + z^2 - 1)/2pz]$

Lamb Darkening

- Most star are not uniform source. Their brightness peak at their center.
- There are different model.
  - linear
  - quadratic
  - Non-linear

Lamb Darkening

- $I(r)$ is the specific intensity as a function of r or μ with $I(0) = 1$. $\mu = \cos(\theta)$
- linear: $1 - c_1 (1-\mu)$
- quadratic: $1 - c_1 (1-\mu) - c_2(1-\mu^2)$
- Non-linear: $I(r) = 1 - \sum^4_{n=1} c_n (1-\mu^{1/2})$



Adapted from: Kaisey Mandel and Eric Agol 2002 ApJ 580 L171

Lamb Darkening

- Inculde the effect of lamb darkening with result in uniform source
- First term is the unobscured flux of the star
- Second term is the obscured flux

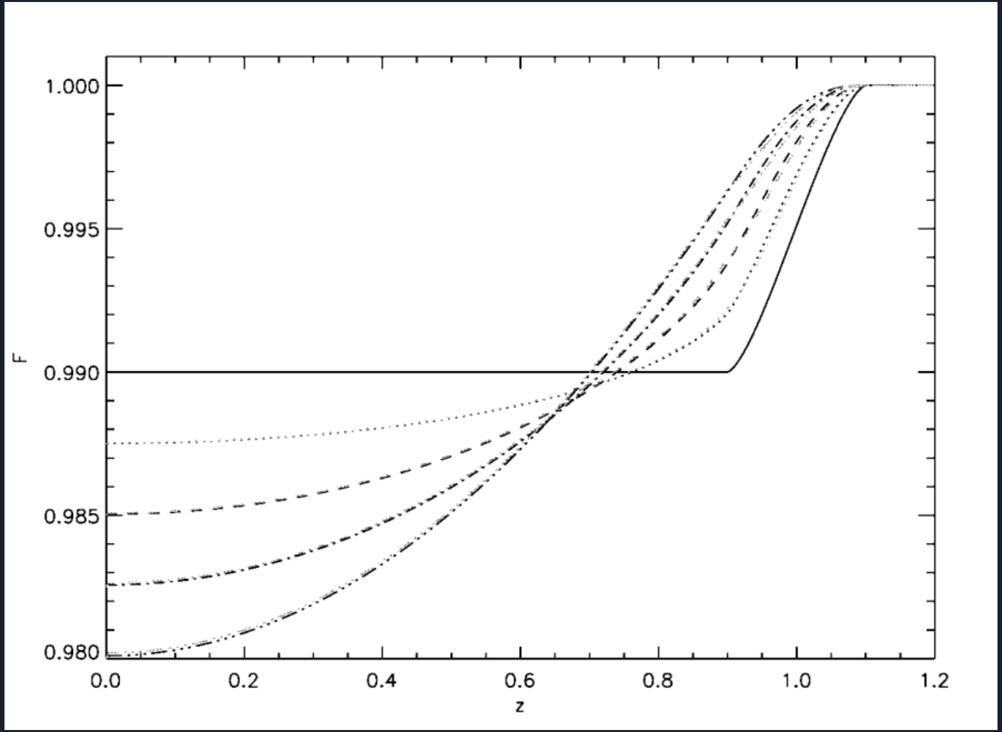
$$F(p, z) = \left[\int_0^1 dr\, 2rI(r)\right]^{-1} \int_0^1 dr\, I(r) \frac{d[F^e(p/r, z/r)r^2]}{dr},$$

Transit model

- $F(p,z)$ can be solved analytically.
- Result is too complicated to be included.
- The resulting light curve depend on few parameters, p, w, c_n

Theoretical light curve

- The solid line is result of uniform source.
- The dashed and dotted line are result when considering non-linear limb darkening with different c_n



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Case study: A sub-Mercury-sized exoplanet

- Kepler-37, a sun-like star of three orbiting planets
  - Cooler than the sun (Cool main-sequence star)
- Kepler-37b, the innermost smallest planets, sub-Mercury-sized
- Kepler-37c
- Kepler-37d

Barclay, T., Rowe, J., Lissauer, J. et al. A sub-Mercury-sized exoplanet. Nature 494, 452–454 (2013). https://doi.org/10.1038/nature11914

Determining stellar properties of Kepler-37

- 978 days by Kepler in the photometric time series data of Kepler-37
- Initial values of Kepler-37 stellar properties were fitted by observed echelle spectra with synthetic spectra *(Credit: Valenti and Fischer, 2005)*

- Detect solar-like oscillations
  - Caused by turbulent convection in its outer layers, like our sun
  - Mixed pressure-gravity modes that are excited over a range in frequency, with the amplitudes roughly following a bell-shaped distribution *(Credit: Wiki)*
  - The frequency separation can be used to precisely determine the mass and radius of star

Solar-like oscillations

- Short-cadence (1 minute time resolution) flux time series of Kepler-37
- Remove the transit effects
- Observe the frequency modes and measure the frequency separation

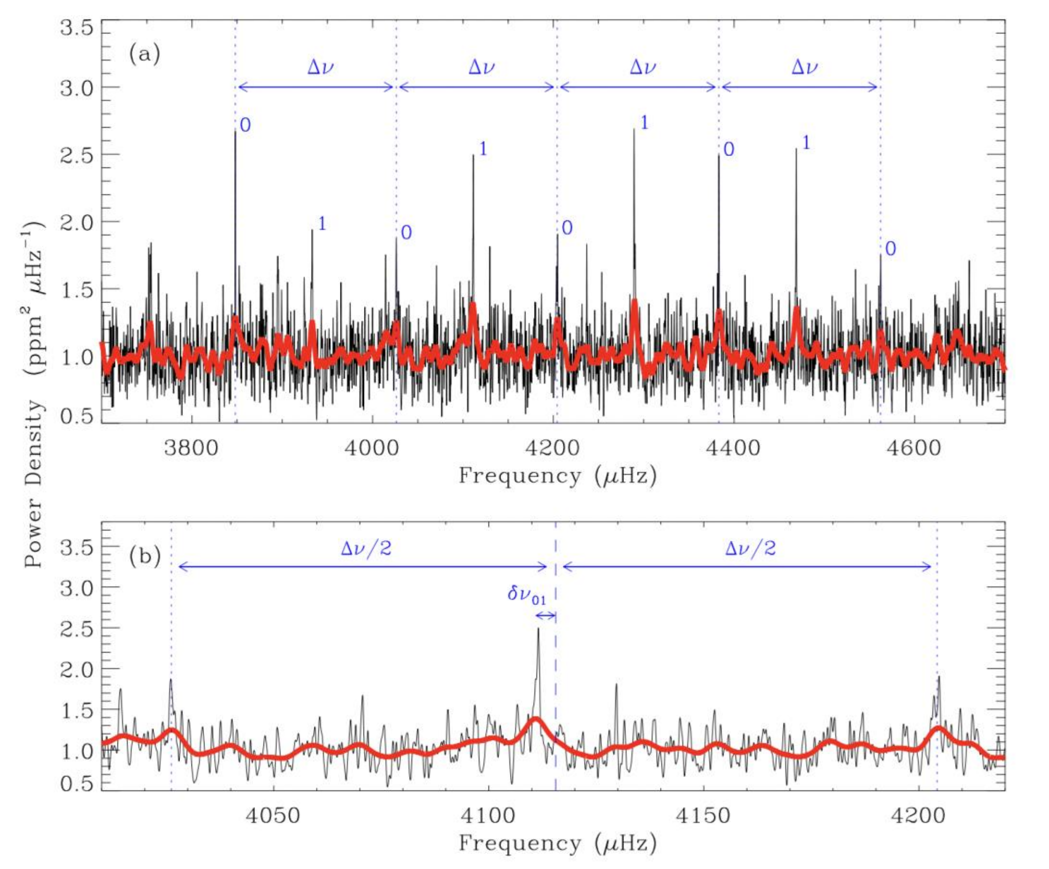


Figure:

(a) Smoothed power density spectrum (1-minute sampling for 15 months from March 2010 to June 2011).
(b) Zoom-in view of one of the large frequency separation at ~4000 - 4200 μHz.

- Notations
  - Radial modes: l = 0
  - Dipole modes: l = 1
  - Δv: Large frequency separation
  - δv: Small frequency separation

(Credit: Supplementary information section)

Calculations of Kepler-37 Mass and Radius

Scaling relations of solar oscillations

$$\begin{cases} \nu_{max} \propto \dfrac{g}{\sqrt{T_{eff}}} \\ \Delta\nu \propto \sqrt{\dfrac{M}{R^3}} \end{cases}$$

- **g:** Surface gravity of star
- **T_eff**: Effective temperature
- These values are determined using other methods

Upon using the Newtonian gravitation relation, stellar mass and radius can be expressed as

$$\begin{cases} M \propto \dfrac{\nu_{max}^3 T_{eff}^{3/2}}{\Delta\nu^4} \\ R \propto \dfrac{\nu_{max} T_{eff}^{1/2}}{\Delta\nu^2} \end{cases}$$

Final estimated values

- Stellar radius: $0.772 \pm 0.026\ R_\odot$
- Stellar mass: $0.803 \pm 0.068\ M_\odot$

Fitting of transiting planet model

- Markov Chain Monte Carlo (MCMC)
  - Sample the planet parameters to account for correlated variables
  - Well suited to the high-dimensional parameter spaces for the multiple-planet systems
- Uncertainity in orbital parameters -> probability distributions of parameters
  - Starts with a known function proportional to the probability distribution (prior)
  - MCMC to sample and integrate the probability distribution
  - Update the function by Bayes theorem (posterior)

$$p(\boldsymbol{x}|\boldsymbol{d}) = \frac{p(\boldsymbol{d},\boldsymbol{x})}{\int p(\boldsymbol{d},\boldsymbol{x})p(\boldsymbol{x})d\boldsymbol{x}} = \frac{p(\boldsymbol{x})p(\boldsymbol{d}|\boldsymbol{x})}{\int p(\boldsymbol{d},\boldsymbol{x})p(\boldsymbol{x})d\boldsymbol{x}}. \tag{1}$$

- Notations
  - **p(d,x):** joint probability of observed data and x
  - **p(x|d):** conditional probability of x given observed data

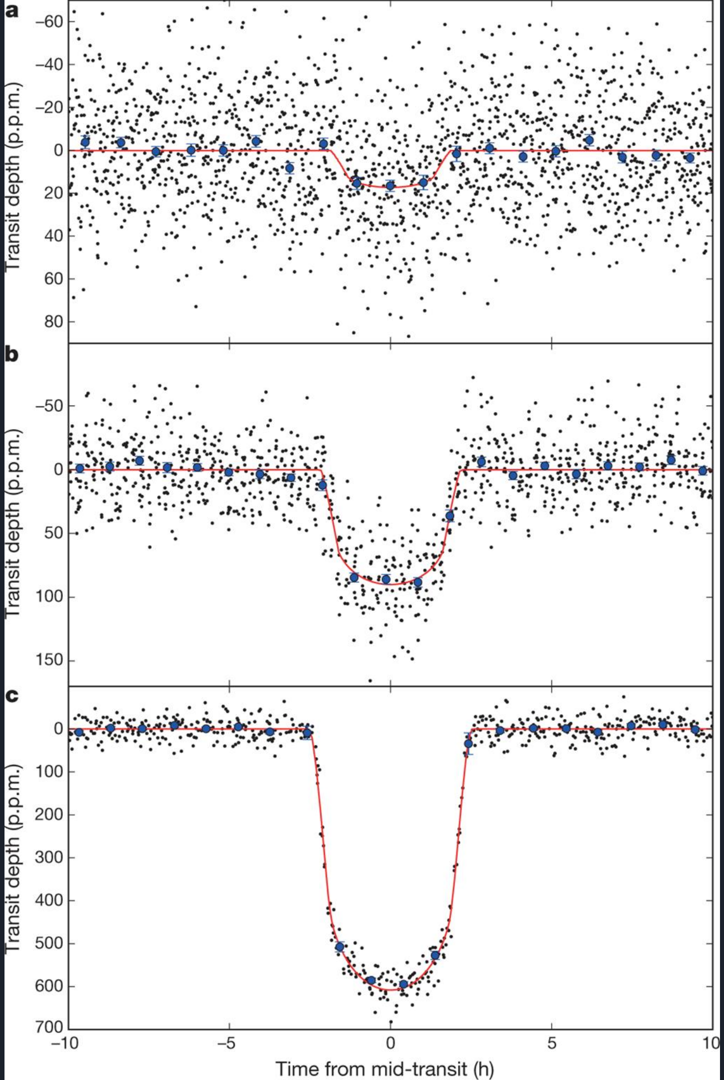
The transits of planets Kepler-37b (a), Kepler-37c (b) and Kepler-37d (c).

The photometric light curve is phase-folded on the orbital period making observed data as a function of orbital phase.

Individual data points are shown as black dots.

The blue dots show the data binned, with 90, 50 and 30 individual data points making up each binned point in a–c, respectively.

The best-fitting transit model from MCMC analysis is the red line.



Radius of the three exoplanets

- Upon fitting the transit model, the ratio of planet radius to stellar radius is yielded

	Kepler-37b	Kepler-37c	Kepler-37d
Radius (in Earth radius)	0.303	0.742	1.99

Implications and conclusion

- The detection of Kepler-37b is remarkable given that this transit signal would be detected in the data of fewer than 0.5% of the stars observed by Kepler.
- Planet occurrence may increase exponentially with decreasing planet size.
- Further updates the size distribution of exoplanets
  - More precise calculations on likelihood of solar system-like planetary systems / earth-like planets
  - Helps pinpoint extraterrestial life search

Reference

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